



Students' Performance on Equations and Story Problems Involving Algebraic Proportions



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Introduction

- Popular perception: story problems are difficult [1]
- In some cases, story problems may be less difficult than equations [2,3]

Simple Algebra Problems: Verbal Advantage^[2, 3]

- Linear equations with one reference to a variable
- Stories support informal strategies

Ted works as a waiter. He worked 6 hours in one day and also got \$66 in tips. If he made \$81.90 that day, how much per hour does Ted make?

$$6 \cdot x + 66 = 81.90$$

Complex Algebra Problems: Symbolic Advantage^[3]

- Equations with multiple variables or references to a variable
- More challenging to solve informally

Roseanne just paid \$38.24 for new jeans. She got them at a 15% discount. What was the original price?

$$X - 0.15X = 38.24$$

Is the tradeoff between simple and complex problems due to the **difficulty** of the problem or the use of **informal strategies**?

Method

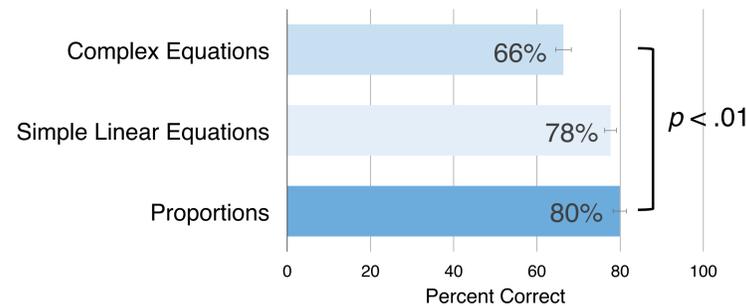
- $N = 351$ students drawn from 19 algebra I classes
- Students completed six types of problems during class time

Problem Type	Instructions	Example
Simple linear equations	Solve for x .	$x - 10 = 4$
Complex linear equations	Solve for x .	$\frac{1}{2(x-6)} = 1 - 3x$
Proportions equations	Solve for x .	$\frac{4}{3} = \frac{8}{x}$
Proportions story problems	Solve for the missing value.	A family used 15 gallons of paint to paint all 6 rooms in their house. How many gallons of paint will they need to paint 8 rooms?
Writing from number sentences	Write the following as a proportion.	5 is to 6 as x is to 12.
Writing from stories	Write the following as a proportion.	A student can read 14 pages in 30 minutes. How many can he read in 45 minutes?

Results

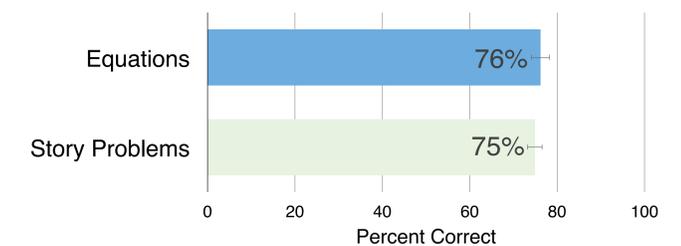
How Difficult Are Proportions?

- Significantly easier than complex linear equations ($F(1, 305) = 53.79, p < .001$)
- Not significantly different from simple equations ($F(1, 305) = 1.66, p = .20$)



Verbal or Symbolic Advantage?

- Neither! No significant difference in performance on story problems or equations containing proportions ($t = 0.73, p = .47$)



Component Skills of Story Problems

- Writing from stories and solving equations predict story-problem success on proportions problems
- Possible ceiling effect on writing from number sentences

	B	SE	β
Writing from number sentences	0.09	0.07	0.07
Writing from stories	0.12*	0.06	0.11
Solving proportions equations	0.62**	0.06	0.51

$R^2 = .324, * p < .05, ** p < .01$

- Error analyses further support this finding
 - 35% of errors: Mistakes in translating from stories
 - 58% of errors: Mistakes in solving proportions equations
 - 6% of errors: Both mistakes

Story Problem Strategies

- Story problem solution strategies coded as in previous work [2]
- Most popular strategy: Symbol manipulation
- Few informal strategies: Less than 10% of problems

Strategy	Frequency	Accuracy
Unwind	7.4%	87%
Guess & test	1.1%	25%
Symbol manipulation	79.2%	86.1%
Answer only	2.9%	20%
Unknown	1.0%	10%
Linear equation	0.8%	0%
Total with response	92.3%	81.9%
No response	7.7%	0%
Total	100%	75.5%

Conclusions

- No symbolic or verbal advantage for proportions
 - **Implication:** Previous findings on the verbal advantage can be attributed to strategy choice and not difficulty
- Very few informal strategies on proportions story problems
 - **Implication:** Instruction should help students use contexts in story problems to check their work
- Two skills (translating from stories & solving equations) contribute to story-problem solving success
 - **Implication:** Instruction must address both skills

References

- [1] Nathan, M.J., & Koedinger, K.R. (2000). Teachers' and researchers' beliefs about the development of algebraic reasoning. *Journal for Research in Mathematics Education*, 168-190.
- [2] Koedinger, K. R., & Nathan, M. J. (2004). The real story behind story problems: Effects of representations on quantitative reasoning. *The Journal of the Learning Sciences*, 13(2), 129-164.
- [3] Koedinger, K. R., Alibali, M. W., & Nathan, M. J. (2008). Trade-offs between grounded and abstract representations: Evidence from algebra problem solving. *Cognitive Science*, 32(2), 366-397.

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