



Does Solving for X Help You Solve in Context?

Investigating component skills that contribute to word-problem solving performance in algebra

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Introduction

Word problems involve two component skills. [1,2]

1. Comprehension phase: Translating from words to symbols
2. Solution phase: Solving number problems

Conflicting factors suggest word problems may be easier or more difficult than symbol problems

- Students may not know which operations to use; matching words to symbols can be challenging [3]
- + Students can draw on real-life experiences [4]
- + A productive strategy is to construct a model of the situation [5]

In algebra, challenge depends on the difficulty of the math.

Simple Algebra Problems: Verbal Advantage

(Koedinger & Nathan, 2004)[6]

- Linear equations with only one reference to a variable
- Students perform better on word problems than on matched symbolic problems
- Stories in word problems support informal strategies: guess & test, unwinding

Ted works as a waiter. He worked 6 hours in one day and also got \$66 in tips. If he made \$81.90 that day, how much per hour does Ted make?

$$6 \cdot x + 66 = 81.90$$

Complex Algebra Problems: Symbolic Advantage

(Koedinger, Alibali, & Nathan, 2008)[7]

- Equations with multiple variables or references to a variable
- Students perform better on symbolic problems than on matched word problems
- With multiple references to unknowns, word problems are more challenging to translate into symbols

Roseanne just paid \$38.24 for new jeans. She got them at a 15% discount. What was the original price?

$$X - 0.15X = 38.24$$

Method

- 345 students drawn from algebra classes throughout Midwestern US
- Sample: 16 proportions problems completed during class time
- Word & Number problems matched for equation structure and size of numbers

Translating Problems

Write a proportion to represent the words. You **do not** need to solve the proportions.

x is to 24 as 5 is to 6.

A student can read 14 pages in 30 minutes. How many can he read in 45 minutes?

Word Problems

Solve the problem.

A family used 15 gallons of paint to paint all 6 rooms in their house. If they move to a house with 8 rooms, how many gallons of paint will they need?

Symbolic Problems

Solve the problem.

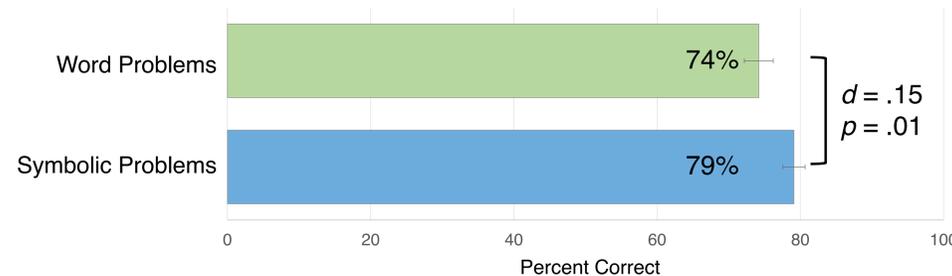
$$5/6 = 2/m$$

Conclusions

- Symbolic advantage for proportions
 - Better performance on number problems than word problems with proportions
- More refined understanding of complexity in algebra problems
 - Instead of focusing on steps in algorithm, perhaps focus on number of relations in problem setup
 - **Implication:** Proportions should be taught as complex algebra problems
- Two skills contribute to word-problem solving success
 - Translating from words to symbols
 - Solving number problems
 - **Implication:** Instruction must address both skills

Proportions: Simple or complex?

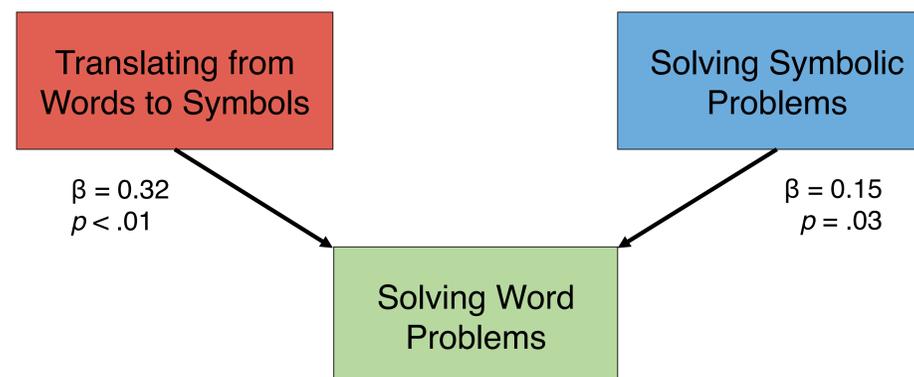
- Algebraically, proportions are simple, 2-step linear equation problems
- Proportions involve relations among multiple values, as in the complex algebra problems



Finding:

- Better performance on symbolic problems than word problems, $t(344) = 2.52$, $p = .01$
- **Symbolic advantage:** similar to complex algebra problems

Which component skills contribute to success?



* Regression controlling for prior math knowledge, $\beta = 0.17$, $p = .01$

Future Directions

- Sample includes two types of translating problems:
 - Translating from number sentences
 - Translating from cover stories
- Analyze these types as separate component skills
- Conduct more detailed analyses of student errors
 - Do students make translation or arithmetic errors?
- Investigate classroom-level effects using multilevel modeling
- Instructions: use proportions in translating and solving
 - Future work could mix different algebraic structures and avoid prompting which equation structures to use.
- Heuristics and informal strategies for proportions
 - Can we teach students to use unwinding, guess & test, or other informal strategies on proportions word problems?

References

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